**Benha University**
**Faculty of Engineering- Shoubra**
**Electrical Engineering Department**
**2nd Year electrical power**

**Final Term Exam**
**Date: 16th of January 2012**
**Mathematics 3a**
**Duration : 3 hours**

- Answer all the following question
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:4
- Total Mark: 80 Marks

تنبيه: مراعاة إجابة كل جزء فى ناحية مستقلة

1-a)Find Fourier series for the function $f(x) = x,\ 0 \le x \le \pi/2$, period $= 2\pi$ in even cosine harmonic and find $\sum_{m=1}^{\infty} \frac{1}{(2m-1)^4}$.

1-b) Expand into complex Fourier series the periodic function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ of period $2\pi$

**(25 marks)**

2) Solve the linear programming problem:

Max $f = 2x + y + 4z$, Subject to: $x + y + 2z \le 20$, $2x + 3y + 2z = 18$, $x + 2y + 2z \ge 6$, $x,y,z \ge 0$

**(20 marks)**

## Probability and Statistics

3-a) Suppose that there is a test for a certain disease. If a person with the disease takes the test, then the test will come back positive 95% of the time (like most medical tests it isn't foolproof). On the other hand, the test will show that you are positive 3.5% when you do not have the disease.

(i) Given that I test positive, what is the chance that I have the disease?

(ii) Given that someone in the sample tests negative, what is the probability that (s)he really does not have the disease?

3-b) A coin is biased so that heads is twice the tails for three independent tosses of the coin, **find**

(i) The probability of getting at most two heads.

(ii) C.d.f. of the random variable X, and use it to find $P(1 < X \le 3)$; $P(X > 2)$.

**(15 marks)**

4-a) Let X and Y denote the amplitude of noise signals at two antennas. The random vector (X, Y) has the joint pdf $f(x, y) = ax\,e^{-ax^2/2}\ by\,e^{-by^2/2}$ $x>0, y>0, a>0, b>0$ , **find**

(i) P[ X>Y]

(ii) Standard diviation of X

4b-i) Derive m.g.f. for gamma distribution, then deduce $\mu_r'$, r = 0,1,2,3

4b-ii) Given a bag containing 3 black balls, 2 blue balls and 3 green balls, a random sample of 2 balls is selected. Given that X is the number of black balls and Y is the number of blue balls, find the joint probability distribution of X and Y and Cov(X,Y).

**(20 marks)**

Good luck BOARD OF EXAMINERS: ***Dr. Ibrahim Sakr & Dr. Khaled El Naggar***

**Model answer**

3a) Let disease: D, and doesn`t have disease:D` , P: positive , N: negative such that P(D) = P(D`) = 0.5 and P(P/D) = 0.95, therefore P(N/D) = 0.05, P(P/D`) = 0.035, thus P(N/D`) = 0.965, so P(D/P) = [P(P/D)P(D)]/P(P) = (0.95)(0.5)/[0.95(0.5) + 0.035(0.5)] and P(D`/N) = P(N/D`) P(D`)/P(N) = 0.965(0.5)/[0.05(0.5) +0.965(0.5)

3b-i) P(H) = 2 P(T), therefore P(H) = 2/3 = P, and $P(H \le 2) = \sum_{x=0}^{2} {}^3c_x (2/3)^x (1/3)^{3-x}$

ii) $F(x=0) = {}^3c_0 (2/3)^0 (1/3)^3$, $F(x=1) = \sum_{x=0}^{1} {}^3c_x (2/3)^x (1/3)^{3-x}$, $F(x=2) = \sum_{x=0}^{2} {}^3c_x (2/3)^x (1/3)^{3-x}$,

$F(x=3) = \sum_{x=0}^{3} {}^3c_x (2/3)^x (1/3)^{3-x}$, $P(1 < X \le 3) = F(x=3) - F(x=1) = \sum_{x=0}^{3} {}^3c_x (2/3)^x (1/3)^{3-x} - \sum_{x=0}^{1} {}^3c_x (2/3)^x (1/3)^{3-x}$, $P(X > 2) = F(x=3) - F(x=2) = \sum_{x=0}^{3} {}^3c_x (2/3)^x (1/3)^{3-x} - \sum_{x=0}^{2} {}^3c_x (2/3)^x (1/3)^{3-x}$

4a-i) $P(X>Y) = \int_0^\infty \int_y^\infty axe^{-ax^2/2} bye^{-by^2/2}\, dx\, dy = \int_0^\infty -e^{-ax^2/2}\Big|_y^\infty bye^{-by^2/2}\, dy = \int_0^\infty e^{-ay^2/2} bye^{-by^2/2}\, dy$

$= \int_0^\infty bye^{-(a+b)y^2/2}\, dy = -\frac{b}{a+b} e^{-(a+b)y^2/2}\Big|_0^\infty = \frac{b}{a+b}$, $f_1(x) = \int_0^\infty [axe^{-ax^2/2} bye^{-by^2/2}]\, dy$,

$E(X) = \int_0^\infty x f_1(x)\, dx$ and $E(X^2) = \int_0^\infty x^2 f_1(x)\, dx$, therefore $Var(X) = E(X^2) - [E(X)]^2$

4b-i) The moment generating function can be expressed by

$$E(e^{tx}) = \int_0^\infty e^{tx} \left(\frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}\right) dx = \frac{\beta^\alpha}{\Gamma\alpha} \int_0^\infty x^{\alpha-1} e^{-(\beta-t)x} dx$$

Put $(\beta - t)x = y \Rightarrow dx = \frac{dy}{\beta - t}$, thus $E(e^{tx}) = \frac{\beta^\alpha}{(\beta-t)^\alpha \Gamma\alpha} \int_0^\infty y^{\alpha-1} e^{-y} dy = \frac{\beta^\alpha}{(\beta-t)^\alpha}$, $\mu_0' = 1$, $\mu_1' = E(X) = \alpha/\beta$, $\mu_2' = E(X^2)$ and $\mu_3' = \frac{d^3}{dt^3}\left[\frac{\beta^\alpha}{(\beta-t)^\alpha}\right]\Big|_{t=0}$

4b-ii) B:Black, b: Blue, G: green

| X \ Y | 0 | 1 | 2 | $f_1(x)$ |
|---|---|---|---|---|
| 0 | P(GG) = 0.1071 | 2P(BG) = 0.3214 | P(BB) = 0.1071 | 0.5356 |
| 1 | 2P(bG) = 0.2143 | 2P(Bb) = 0.2143 | 0 | 0.4286 |
| 2 | P(bb) = 0.0357 | 0 | 0 | 0.0 357 |
| $f_1(x)$ | 0.3571 | 0.5357 | 0.1071 | 1 |

$E(Y) = 0(0.5356) + 1(0.4286) + 2(0.0357) = 0.5$, $E(X) = 0(0.3571) + 1(0.5357) + 2(0.1071) = 0.75$, $E(XY) = 1(0.2143) = 0.2143$, therefore $Cov(X,Y) = E(XY) - E(X) E(Y) = -0.1607$